TIME, SPEED AND DISTANCE

1. Time, speed and distance are proportionally related to each other.

(a) *When time is constant, distance covered is directly proportional to the speed.*

Ex. A train leaves Delhi for Mumbai, a distance of 1600 km and at the same time another train leaves Mumbai for Delhi. These trains meet at Bhopal at a distance of 600 km from Delhi. What is the ratio of their speed?

Sol. Ratio of speed = ratio of distance covered

$$\frac{\text{Speed A}}{\text{Speed B}} = \frac{600}{(1600-600)} = \frac{600}{1000} = \frac{3}{5}.$$

(b) *When distance is the same, speed is inversely proportional to time.*

Ex. If I travel at $\frac{4}{5}$ of my speed, I reach my office 6 minutes late. What is the original duration of time I take to reach office?

2. Relative speed

(a) When traveling in opposite direction.

Relative speed = $V_1 + V_2$

(b) If moving in same direction.

Relative speed = $V_1 - V_2$

3. Train crossing

If L_1 and L_2 are the lengths of two trains and they move at speeds V_1 and V_2, respectively.

Then time taken by them to cross each other = $\frac{L_1 + L_2}{\text{Relative speed}}$.

4. Boats and Streams

(a) Down stream speed (D) = Speed of Boat + Speed of Stream

(b) Upstream speed (U) = Speed of Boat – Speed of Stream

Speed of boat = $\frac{D+U}{2}$

Speed of stream = $\frac{D-U}{2}$

5. Circular motion

(a) Time taken to meet for the first time, if two runners are running on the same circular track

$= \frac{\text{Track length}}{\text{Relative speed}}$

(b) Number of times two runners meet on the circular track = Number of rounds gained by faster runner over the slower one.

(c) If ratio of speeds of two runners running in circular track is x : y, they will meet at the starting point again in

|x – y| time (if running in the same direction)

(x + y) time (if running in the opposite direction)

Ex. Two runners are running at speeds 2 m/s and 5 m/s on a circular track. How many times will they cross each other before finally meeting at the starting point?

(a) When in same direction

(b) When running in opposite direction

6. Average speed:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Case A: If distance traveled is equal

Average speed = $\frac{2V_1V_2}{V_1+V_2}$ [For two different speeds V_1 and V_2]

OR Average speed = $\frac{3V_1V_2V_3}{V_1V_2+V_2V_3+V_3V_1}$ [For three different speeds V_1, V_2 and V_3].

Case B: When time taken is the same

Average speed = $\frac{V_1+V_2}{2}$

OR Average speed = $\frac{V_1+V_2+V_3}{3}$

Where V_1, V_2, V_3 are speeds for different distances covered in equal intervals of time.

Important Fact:

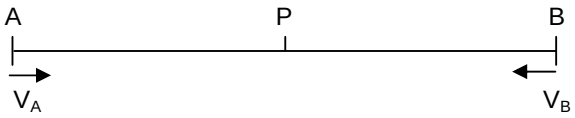
Average speed can never be double or more than double of any of the original speeds.

Ex. A formula one driver covers some distance at a speed of 60 mph. With what speed should he return, so that the average speed for the total journey will be 120 mph?

(1) 180 mph (2) 120 mph (3) 150 mph (4) Impossible

7. If two persons start moving towards each other from points A and B simultaneously at speeds V_A and V_B. After meeting, they take t_A and t_B minutes to reach the other ends (B and A) respectively then

$$\frac{V_A}{V_B} = \sqrt{\frac{t_B}{t_A}}$$



CLOCKS

Problems on clock: These are also based on circular motion concept.

Speed of the minute hand = $\frac{360^0}{60} = 6^o$/min

Speed of the hour hand = $\frac{360^0}{60 \times 12} = \left(\frac{1}{2}\right)^o$ /min

Relative speed of the minute and the hour hand = $\left(6 - \frac{1}{2}\right)^0$ /min = $\left(\frac{11}{2}\right)^o$ /min

<u>To calculate the angle between the two hands when time is given</u> e.g find the angle between two hands at 5:30:

We can apply the following direct formula to find the angle between the hands:

$$30° \times (\text{hr} - \frac{\text{min}}{5}) \pm \frac{\text{min}}{2}$$

[(+) if hr hand is ahead of min hand and (–) if it is before min hand.]

<u>To calculate the time when both the hands will be at some angle</u> e.g At what time between 4 and 5 o'clock are the hands of the clock together?

If between n and (n +1) o'clock, the two hands are together at an angle θ then required time = $\frac{2}{11}$ [$n \times 30^o \pm \theta$] mins

e.g.

Required time = $\frac{2}{11}$ [$(n) \times 30^o \pm \theta$]

Here $\theta = 0^o$ (Hands of clock are together) and n = 4

$\therefore$ Required time = $\frac{240}{11} = 21\frac{9}{11}$ min.

<u>Overall Gain/loss per day by a watch,</u> After every $65\frac{5}{11}$ min. = $\frac{720}{11}$ min. the two hands will coincide. If the hands of a clock coincide every 'x' min, then gain/loss is given by

$$\left(\frac{720}{11} - x\right) \times \frac{60 \times 24}{x}$$

[If answer is (+) then there will be gain and if (–) then there will be loss.]

Problems on speed (slow/fast) of the hands of a clock:

Two clocks show the same time i.e. 9.00 am on 23rd April. The first clock will gain 10 min every 2 hours and second clock loses 10 min every hour. When will they show the same time again?

Hint: These types of problems can be solved with the help of the concept of circular motion i.e. they will have a gap of 12 hrs.

In 1 hour, there will be a difference of 15 min in there time (since the first one gains 5 min and second one looses 10 min).

To get separated by 12 hrs (12×60 min), time taken = $\frac{12 \times 60}{15}$ = 48 hrs.

Important fact:

(1) Both hands meet after every $65\frac{5}{11}$ minutes.

(2) In 12 hours hands of a clock coincide 11 times.

(3) In 12 hours, hands of a clock come opposite to each other 11 times.

(4) In 12 hours hands of a clock at right angles to each other or at any other angle other than 180^o = 22 times.

Questions on Races *e.g In one kilometer race, A beats B by 36 metres or 9 seconds. Find A's time over the course.*

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Important formulae:

(a) Winner's distance = Length of race

(b) Loser's distance = Winner's distance – (beat distance + start distance

(c) Winner's time = Loser's time – (beat time + start time)

(d) $\frac{\text{Winner's time}}{\text{Loser's distance}} = \frac{\text{Loser's time}}{\text{Winner's distance}}$

$= \frac{\text{beat times} + \text{start time}}{\text{beat distance} + \text{start distance}}$

(e) If a race ends in a deadlock, i.e. both reach the winning post together then beat time = 0 and beat distance = 0.